Magnetic

Magnetic flux density or magnetic field (B)

The total no. of magnetic lines of force passing normally per unit surface area is known as magnetic flux density.

Magnetic flux density (B) = $\frac{\text{Total magnetic lines of force or magnetic flux } (\phi)}{\text{Area } (A)}$

or, $\boxed{B = \frac{\phi}{A}}$

Preservator quality- It is a vector quantity. It's SI unit is Tesla (T) or weber / meter^2 (Wb/m^2) or $\frac{N}{Am}$.
It's CGS unit is Maxwell / cm^2
It's dimensional formula is $\frac{[MLT^{-2}]}{[A][L]}$
$= [ML^0T^{-2}A^{-1}]$

Oersted's experimental result-
Oersted's experimental result states that, "Whenever current flows through the conductor magnetic field is produced around it which lasts as long as the current flows through it. In other words electric field produces magnetic field.

* Ways to determine the direction of deflection of compass needle when the current flows through the conductor.

a) Snow-rule (South-north over west)-
According to this rule if the current flowing through the conductor is from south to north direction,

the north pole of the compass needle gets deflected in west direction.

b) Amper's rule -

This rule relates the current flowing through the conductor with the swimmer swimming According to Amper's rule if the current flowing through the conductor is represented along the feet to the head of the swimmer. Then the north pole of the compass needle gets deflected in that direction in which the left hand of the swimmer bends.

* Ways to determine the direction of magnetic lines of force when current flows through

a) Straight conductor -

The following rules are used to obtain the direction of magnetic lines of force when current flows through straight conductor:

i) Right hand thumb rule -

If the current flowing through the straight conductor is represented by the thumb right hand then the remaining curled finger represent the direction of magnetic lines of force which are circular. And the tangent drawn to the line of force represents the direction of magnetic field.

ii) Screw rule -

Screw has 2 types of motion i.e. linear and circular. If the linear motion of the screw represents the direction of current flowing through the conductor then the circular motion represents the direction of magnetic lines of force.

b) ~~a~~ solenoid -

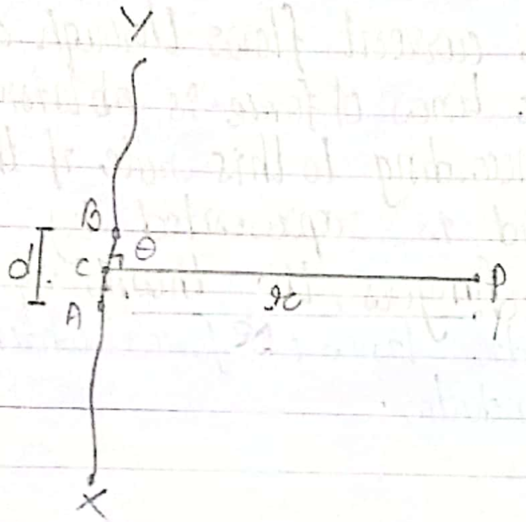
When a current flows through a solenoid the direction of magnetic lines of force is obtained using right hand first rule. According to this rule if the current flowing through the solenoid is represented by fist of right hand or the curled fingers, the thumb ~~of~~ represents the direction of magnetic lines of forces which are parallel with in the range of conductor.

Flemmings left hand rule -

Flemmings left hand rule is used to ~~affer~~ obtain the direction of force acting on conductor carrying current or the charge (positive) in the presence of external magnetic field. According to flemmings left hand rule when three fingers of the left hand i.e the thumb, index and middle fingers are held mutually perpendicular to each other such that the middle finger represents the direction of motion of positive charge or the current flowing through the conductor and index finger, direction of magnetic field then the thumb gives the ~~fore~~ direction of force acting.

Biot and Savart's ~~law~~ rule -

Biot and Savart's ~~law is~~ rule is used to obtain the value of flux density at any specified point with in the magnetic field of the conductor (both symmetrical and asymmetrical) carrying conductor.



let us consider XY is a conductor through which current I is flowing. As the current flows to the conductor magnetic field is produced. Let P is the point within the magnetic field of the conductor where the flux density is to be obtained. let AB = dL, is the small elemental length whose mid point is C. C and P are joined such that CP = r. let θ is the angle betn the elemental length and CP. So, according to Biot and Savart's law, the value of flux density due to the elemental length is small i.e dB and is

a) directly proportional to elemental length of the conductor.

i.e. $dB \propto dl$ —— (i)

b) directly proportional to the current flowing through the conductor i.e.

$dB \propto I$ —— (ii)

5

d) Inversly proportional to square of dist. betn the elemental length and line joining the mid point of elemental length and specified point i.e. $dB \propto \frac{1}{r^2}$ ——— (iv)

c) directly proportional to the sine of the angle betn elemental length and line joining, the midpoint of elemental length & the specified point.

i.e. $dB \propto \sin\theta$ ——— (iii)

Combining all the above relations, we get,

$$dB \propto \frac{I\,dl\,\sin\theta}{r^2}$$

$$\text{Or, } dB = \frac{K I\,dl\,\sin\theta}{r^2} \quad \text{——— (v)}$$

where 'K' is proportionality constant where value depends upon the system of measurement.

In CGS System, $K = 1$

$$\text{So, } dB = \frac{I\,dl\,\sin\theta}{r^2} \quad \text{——— (vi)}$$

In SI system, $K = \frac{\mu_0}{4\pi}$, where μ_0 is the permeability of free space whose value is 4×10^{-7} Henry/meter (H/m).

$$\text{So, } dB = \frac{\mu_0}{4\pi} \frac{I\,dl\,\sin\theta}{r^2} \quad \text{——— (vii)}$$

In vector notation,

$$\left.\begin{aligned} \vec{dB} &= \frac{\mu_0}{4\pi} \frac{I(\vec{dl} \times \hat{r})}{r^2} \\ \text{also, } \vec{dB} &= \frac{\mu_0}{4\pi} \cdot \frac{I\,(\vec{dl} \times \vec{r})}{r^3} \end{aligned}\right\} \quad \text{(viii)}$$

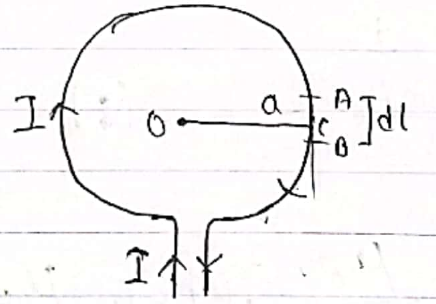
Scanned with CamScanner

So, the total value of flux density due to entire length of the conductor can be obtained by integrating eq^n (vii) under the respective limit whose value depends upon the type of conductor.

i.e. $$B = \int_0^B dB = \int_0^b \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta}{r^2} \quad \text{—— (ix)}$$

* Applications of Biot and Savart's law -

i) Flux density at the centre of circular coil carrying current.



let us consider a circular coil of having centre O and radius 'a' through which current 'I' is flowing. As the current flows through the coil, magnetic field is produced and is to be obtained at the centre 'O'. Let AB = dl is the small elemental length of the conductor whose mid point is 'C'. C and O are joined and is equal to the radius of the coil. The angle bet^n $\vec{dl}$ and $\vec{a}$ is $90°$ as dl is tangent to $\vec{a}$, so, the value of flux density at the centre of coil due to the elemental length according to

Biot Savart's law is

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin 90^\circ}{a^2}$$

$$\text{or, } dB = \frac{\mu_0}{4\pi} \frac{I\,dl}{a^2} \quad \text{———} \quad (i)$$

Then, the total flux due to the entire length of coil is obtained by integrating eqⁿ (i) under the limit 0 to $2\pi a$.

$$\text{i.e. } B = \int_0^B dB = \int_0^{2\pi a} \frac{\mu_0}{4\pi} \frac{I\,dl}{a^2}$$

$$\text{or, } B = \frac{\mu_0 I}{4\pi a^2} \int_0^{2\pi a} dl$$

$$\text{or, } B = \frac{\mu_0 I}{4\pi a^2} \cdot 2\pi a$$

$$\text{or, } B = \frac{\mu_0 I}{2a}$$

If there are 'N' number of such circular coils, then the total flux density becomes,

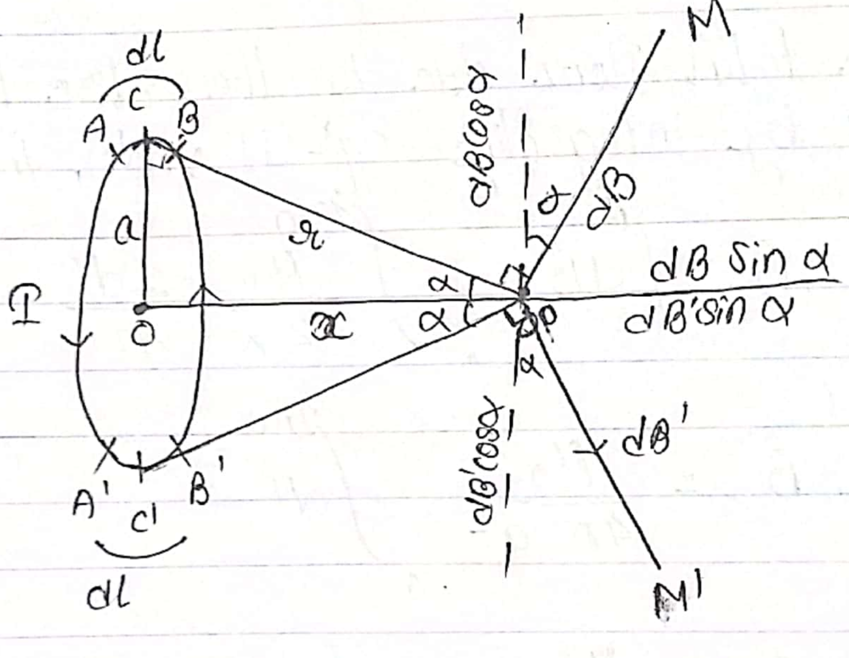
$$\boxed{B = \frac{\mu_0 NI}{2a}}$$

Special case-

I. If the radius of the circular coil is extremely small, the value of flux density is almost infinite at the centre.

II. If the radius of the circular coil is extremely large, the value of flux density is almost zero at the centre

ii) Flux density along the axis of circular coil carrying current.



Let us consider a circular coil of radius 'a' with centre 'O' through which current 'I' is flowing. As the current flows through the coil, magnetic field is produced which is due to the magnetic lines of forces. let 'P' is the point at a distance 'x' from the centre 'O' where the value of flux density is to be obtained. let AB = dL is the small elemental length whose midpoint is 'C'. 'C' and 'P' are joined such that CP = r. The angle between $\vec{dl}$ and $\vec{r}$ is 90° as $\vec{r}$ is tangent to $\vec{dl}$. So the value of flux density due to the elemental length as per Biot and Savart's rule is given as 'P' is

$$dB = \frac{\mu_0}{4\pi} \frac{I dL \sin 90^\circ}{r^2}$$

$$\text{or, } dB = \frac{\mu_0}{4\pi} \frac{I dL}{r^2} \quad \text{——— (1)}$$

Since dB is vector quantity it acts along PM as

$$dB = \frac{\mu_0}{4\pi} \frac{I \vec{dl} \times \vec{r}}{r^3}$$

$\vec{dB}$ ⊥ to the plane which contains $\vec{dl}$ and $\vec{r}$.

let A'B' = dL is the elemental length at the diametrically opposite end where the value of flux density is seen as dB and acts along PM and is given as dB'

On resolving dB and dB' we get dB cos α and dB' cos α respectively as the vertical components which cancels out each other whereas dB sin α and dB' sin α are the horizontal components which acts along the axis.

So the total value of flux density due to the entire length of the circular coil is obtained by integrating $dB \sin\alpha$ under limits 0 to $2\pi a$

i.e. $$B = \int_0^{2\pi a} dB \sin\alpha$$

Or, $$B = \int_0^{2\pi a} \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2} \sin\alpha$$

Or, $$B = \frac{\mu_0 I}{4\pi r^2} \sin\alpha \int_0^{2\pi a} dl$$

Or, $$B = \frac{\mu_0 I}{4\pi r^2} \sin\alpha \, 2\pi a$$

$$\therefore B = \frac{\mu_0 I a}{2r^2} . \sin\alpha \quad \text{--- (ii)}$$

From the geometry of the figure,
In rt. angled $\triangle COP$

$$\sin\alpha = \frac{CO}{CP} = \frac{a}{r} \quad \text{--- (iii)}$$

Also,
$$r^2 = a^2 + x^2$$
Or, $$r = (a^2 + x^2)^{1/2}$$
Or, $$r^3 = (a^2 + x^2)^{3/2} \quad \text{--- (iv)}$$

Using (iii) in (ii) we get,

$$B = \frac{\mu_0 I a}{2r^2} \cdot \frac{a}{r}$$

Or, $B = \dfrac{\mu_0 I a^2}{2r^3}$

Or, $B = \dfrac{\mu_0 I a^2}{2(a^2+x^2)^{3/2}}$

If there are N numbers of such circular coils then the total value of flux density along the axis becomes i.e. $\boxed{B = \dfrac{\mu_0 N I a^2}{2(a^2+x^2)^{3/2}}}$

Special cases:

a) If the radius of the circullar coil is extremely small such that $a^2+a^2 \approx x^2$, then

$$B = \frac{\mu_0 N I a^2}{2(\cancel{a^2}+x^2)^{3/2}}$$

$$= \frac{\mu_0 N I a^2}{2a^3}$$

b) If the specified point lies at the centre of circular coil then,

$$x = 0$$

$$B = \frac{\mu_0 N I a^2}{2a^3}$$

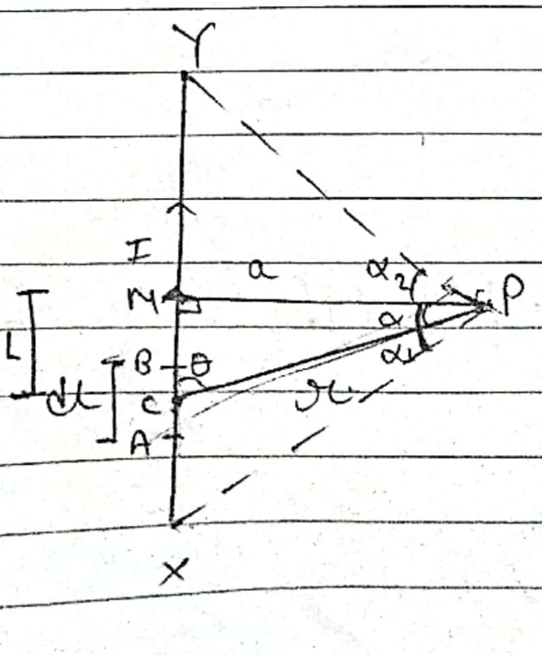
$$\boxed{B = \frac{\mu_0 N I}{2a}}$$

Application

iii) ~~flux density along the axis of swallonoid~~ solenoid carrying current.

iii) Flux density due to long straight conductor carrying current.

Let us consider XY is the long straight conductor through which current 'I' is flowing as the current flows through the conductor ~~fina~~ magnetic field is produced. Let P is the point within the magnetic field of the conductor where the flux density is to be obtained. Let AB = dl is the small elemental length whose mid point is C. C and P are joined such that CP = r. Let θ is the angle between the elemental length and CP.



So, according to Biot and Savart's rule, the value of flux density is given as

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2} \quad \text{---} (1)$$

MP perpendicular on XY is drawn such that,

$MP = a$

let the angle subtended by the elemental length at point 'P' is α. let, MC = l. Let ends 'X' and 'P' as well as 'Y' and 'P' are joined such that angle $\measuredangle MPX = \alpha_1$ and $\measuredangle MPY = \alpha_2$.

From rt. angled ΔCMP

$$\theta + \alpha = 90^\circ$$

$$\text{Or, } \theta = 90^\circ - \alpha$$

Taking sine on both sides,

$$\sin\theta = \sin(90^\circ - \alpha)$$

$$\text{Or, } \sin\theta = \cos\alpha \quad \text{---} (2)$$

also,

$$\cos\alpha = \frac{MP}{CP} = \frac{a}{r}$$

$$\text{Or, } r = \frac{a}{\cos\alpha} \quad \text{---} (3)$$

again,

$$\tan\alpha = \frac{MC}{MP} = \frac{l}{a}$$

Or, $l = a\tan\alpha$

Differentiating both sides w. r. to α, we get

$$\frac{dl}{d\alpha} = a\sec^2\alpha$$

Or, $dl = a\sec^2\alpha \, d\alpha$ —— (4)

Using the values of eq^n (2), (3) and (4) in (1)

$$dB = \frac{\mu_0}{4\pi} \frac{I\, a\sec^2\alpha\, d\alpha \cos\alpha}{\frac{a^2}{\cos^2\alpha}}$$

Or, $dB = \dfrac{\mu_0}{4\pi}\dfrac{I}{a}\sec^2\alpha\cos^3\alpha\, d\alpha$

Or, $dB = \dfrac{\mu_0 I}{4\pi a}\cos\alpha\, d\alpha$ —— (5)

Then the total value of flux density due to the entire length of the conductor is obtained by Integrating eq^n (5) under the limits $-\alpha_1$ to α_2.

i.e. $B = \displaystyle\int_0^B dB = \int_{-\alpha_1}^{\alpha_2} \frac{\mu_0 I}{4\pi a}\cos\alpha\, d\alpha$

$$\text{or, } B = \frac{\mu_0 I}{4\pi a} \int_{-\alpha_1}^{\alpha_2} \cos\alpha \, d\alpha$$

$$\text{or, } B = \frac{\mu_0 I}{4\pi a} \left(\sin\alpha\right)_{-\alpha_1}^{\alpha_2}$$

$$\text{or, } B = \frac{\mu_0 I}{4\pi a} \left(\sin\alpha_2 - \sin(-\alpha_1)\right)$$

$$\text{or, } B = \frac{\mu_0 I}{4\pi a} \left(\sin\alpha_1 + \sin\alpha_2\right)$$

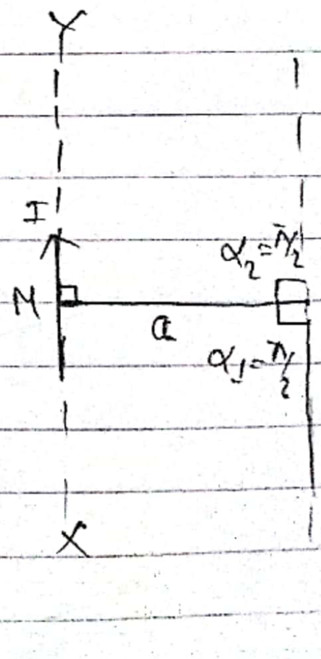
Special cases:

a) If the conductor is infinitely long then,

$$\alpha_1 = \pi/2$$

$$\alpha_2 = \pi/2$$

$$\therefore B = \frac{\mu_0 I}{4\pi a} \left(\sin \pi/2 + \sin \pi/2\right)$$



$$\text{or, } B = \frac{2\mu_0 I}{4\pi a}$$

$$\text{or, } B = \frac{\mu_0 I}{2\pi a}$$

b) If the specified point lies infront of any one end of the Infinite long conductor, (say) infront of end 'x' then,

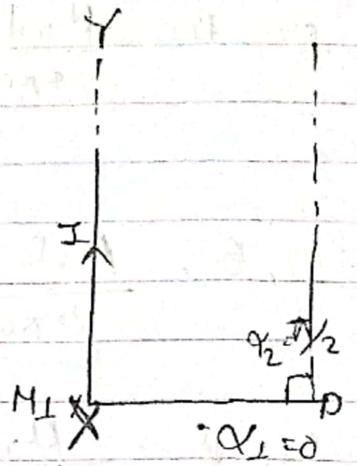
$\alpha_1 = 0$

$\alpha_2 = \pi/2$

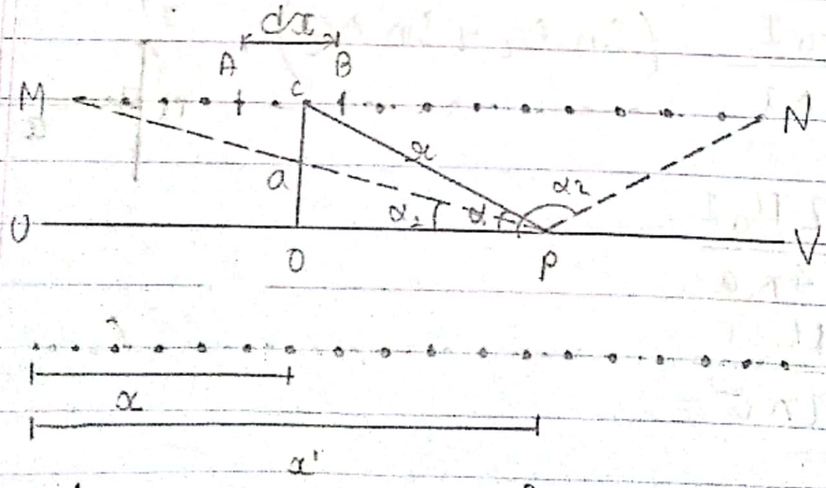
$$\therefore B = \frac{\mu_0 I}{4\pi a}\left(\sin 0^\circ + \sin \pi/2\right)$$

$$\text{or, } B = \frac{\mu_0 I}{4\pi a} \cdot 1$$

$$\text{or, } B = \frac{\mu_0 I}{4\pi a}$$



iv) flux density along the axis of solenoid carrying current.



Let us consider a uniform solenoid has 'n' no. of terms per unit length through which current 'I' is flowing. let, 'a' is the radius of each of the circular coils. let, As the current flows through the circular coils magnetic field is produced along the axis of solenoid. Let P is the point along the axis of solenoid where the flux density is to be obtained. Let, AB = dx is the small elemental length of the solenoid close whose mid point is C. C and P are joint such that CP = r. The total no. of circular coils with in AB is ndx. Then the value of flux density at point 'P' due to the elemental length according to Biot and Savart's rule is

$$dB = \frac{\mu_0 I a}{2r^2} \sin\alpha . ndx \quad \text{—— (1)}$$

where, $B = \frac{\mu_0 I a}{2r^2} \sin\alpha$ is the value of flux density due to a single circular coil along the axis.

Let the ends 'M' and 'P' as well as 'N' and 'P' are joined such that angle $\angle MPO = \alpha_1$ and $\angle NPO = \alpha_2$. Also, $UO = x$ and $UP = x'$

From right angle ΔCOP,

$$\sin\alpha = \frac{CO}{CP} = \frac{a}{r}$$

$$\text{or, } r = \frac{a}{\sin\alpha} \quad \text{—— (2)}$$

And,

$$\tan\alpha = \frac{CO}{OP} = \frac{a}{x' - x}$$

$$\text{or, } x' - x = \frac{a}{\tan\alpha}$$

$$\text{or, } x' - x = a \cot\alpha$$

Differentiating both the sides we get,

Or, $\frac{dx}{d\alpha} = a \operatorname{cosec}^2\alpha$

Or, $dx = a \operatorname{cosec}^2\alpha \, d\alpha$ —— (3)

Substituting the value of eqⁿ (2) and (3) in 1, we get,

$$\therefore dB = \frac{\mu_0 I a}{\frac{2a^2}{\sin^2\alpha}} \sin\alpha \cdot n\, a \operatorname{cosec}^2\alpha \, d\alpha$$

Or; $dB = \frac{\mu_0 n I}{2} \sin^3\alpha \cdot \operatorname{cosec}^2\alpha \, d\alpha$

Or, $dB = \frac{\mu_0 n I}{2} \sin\alpha \, d\alpha$ —— (4)

Then the total value of flux density due to the entire length of the solenoid is obtained by integrationg eqⁿ (4) under the limits α_1 to α_2

i.e. $B = \int_0^B dB = \int_{\alpha_1}^{\alpha_2} \frac{\mu_0 n I}{2} \sin\alpha \, d\alpha$

Or, $B = \frac{\mu_0 n I}{2} \int_{\alpha_1}^{\alpha_2} \sin\alpha \, d\alpha$

Or, $B = \frac{\mu_0 n I}{2} \left(-\cos\alpha \right)_{\alpha_1}^{\alpha_2}$

Or, $B = \frac{\mu_0 n I}{2} \left(-\cos\alpha_2 + \cos\alpha_1 \right)$

or, $B = \frac{\mu_0 n I}{2}(\cos\alpha_1 - \cos\alpha_2)$

Special cases

a) For infinite long solenoid : $\alpha_1 = 0$ and $\alpha_2 = \pi$

$$\therefore B = \frac{\mu_0 n I}{2}(\cos 0^\circ - \cos\pi)$$

$$\text{or, } B = \frac{\mu_0 n I}{2}(1+1)$$

$$\text{or, } B = \frac{\mu_0 n I}{2} \times 2$$

$$\text{or, } \boxed{B = \mu_0 n I}$$

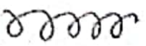
b) If the specified point lies at any one end of infinite long solenoid carrying the ~~length~~ current (say the specified point) lies at end N.

$$\therefore \alpha_1 = \pi/2 \text{ and } \alpha_2 = \pi$$

$$B = \frac{\mu_0 n I}{2}(\cos\pi/2 - \cos\pi)$$

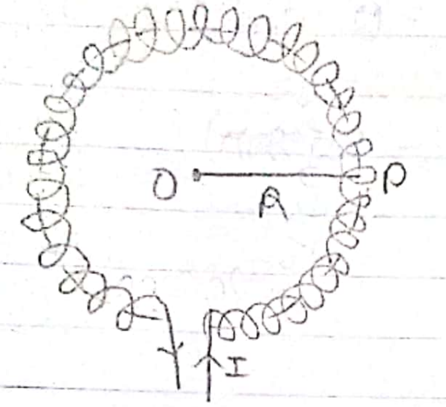
$$\text{or, } B = \frac{\mu_0 n I}{2}(0+1)$$

$$\text{or, } \boxed{B = \frac{\mu_0 n I}{2}}$$

 → Solenoid

 → toroid

v) Flux density due to ~~tota~~ toroid carrying current



let us consider a ~~toroid~~ uniform solenoid having 'n' of turns per unit length is given circular shape to form a toroid of radius 'R'. As the current flows to through the toroid magnetic field is produce which is due to the magnetic lines of forces and is circular. so the total no of circular coils in the given toroid is

$$N = n\,2\pi R$$

$$\text{or, } n = \frac{N}{2\pi R} \quad \text{--- (1)}$$

Since the toroid is formed from the solenoid itself, the value of flux density due to toroid is

$$B = \mu_0 n I$$

$$\text{or, } B = \frac{\mu_0 N I}{2\pi R}$$

21

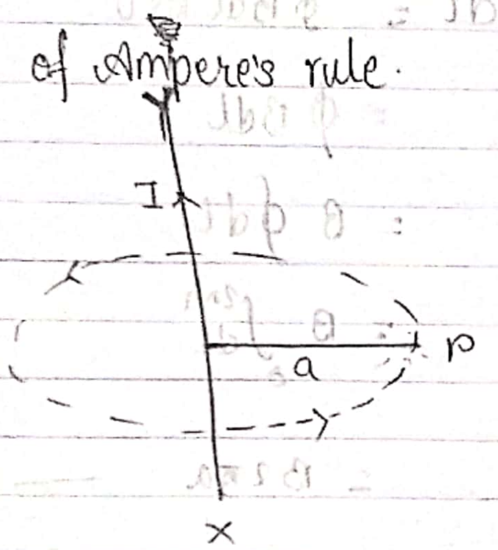
$\oint dl \rightarrow$ line integral magn. - perimeter
$\oiint dS \rightarrow$ Surface " $\rightarrow$ area
$\oiiint dv \rightarrow$ volume " $\rightarrow$ volume

Amperes Circuital law -

Amperes circuital law is used to obtain the value of flux density due to symmetrical conductors carrying current. According to this law, the line integral of flux density along the closed loop is equal to the product of permeability of free space and total current enclosed by the loop.

i.e. $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

* Verification of Ampere's rule.



Verification of amperes' rule -

let us consider a long straight conductor XY through which current I is flowing. As the current flows through the conductor, magnetic field is produced which is due to magnetic lines of forces and is circular. let the magnetic lines of force pass through point p at a distance a from the conductor. So, the value of flux density at point P according to Biot and Savart's rule is

$$B = \frac{\mu_0 I}{2\pi a} \quad \text{---} (1)$$

Then the line integral of flux density along the closed loop gives the value of flux density due to the conductor carrying current

i.e $\oint \vec{B} \cdot \vec{dl} = \oint B\, dl \cos\theta$

But, $\theta = 0°$ as $\vec{B}$ and $\vec{dl}$ are parallel to each other at that point.

$$\oint \vec{B} \cdot \vec{dl} = \oint B\, dl \cos 0°$$

$$= \oint B\, dl$$

$$= B \oint dl$$

$$= B \int_0^{2\pi a} dl$$

$$\oint \vec{B} \cdot \vec{dl} = B\, 2\pi a \quad \text{—— (2)}$$

Using the value of eq^n (1) in (2) we get

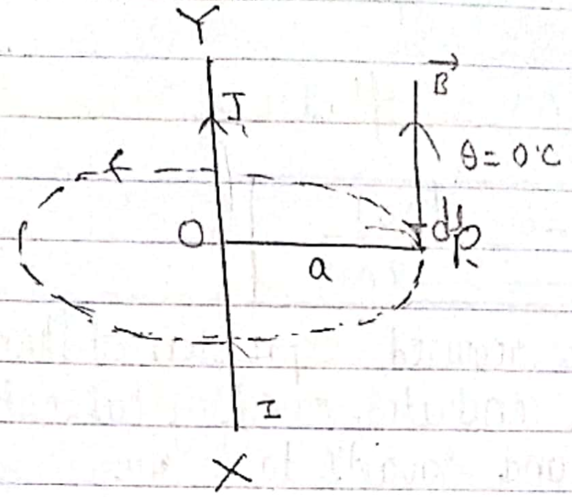
$$\oint \vec{B} \cdot \vec{dl} = \frac{\mu_0 I}{\cancel{2\pi a}} \cdot \cancel{2\pi a}$$

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

Verified

Application of Ampere's circuital law

1) Flux density due to infinite long straight conductor carrying current.



Let us consider a long straight conductor XY through which current I is flowing. As the current flows through the conductor, magnetic field is produced which is due to magnetic lines of force and is circular. Let the magnetic lines of force pass through point 'p' at a distance 'a' from the conductor. ~~So, the value of flux density at point P according to Biot and Savart's rule is~~
Now, from Ampere's l

Then the line integral of flux density along the closed loop is
the value $B = \frac{\mu_0 I}{2\pi a}$ of flux density due to conductor carrying current.

i.e $\oint \vec{B} \cdot \vec{dl} = \oint B\, dl \cos\theta$

But, $\theta = 0°$ as $\vec{B}$ and $\vec{dl}$ are parallel to each other at that point.

$$\oint \vec{B} \cdot \vec{dl} = \oint B\, dl \cos 0°$$

or, $\oint \vec{B} \cdot \vec{dl} = \oint B\, dl$

or, $\oint \vec{B} \cdot \vec{dl} = B \oint dl$

or, $\oint \vec{B} \cdot \vec{dl} = B \int_0^{2\pi a} dl$

$$\therefore \oint \vec{B} \cdot \vec{dl} = B\, 2\pi a \quad \text{---(1)}$$

Now, from Ampere's law, we have,

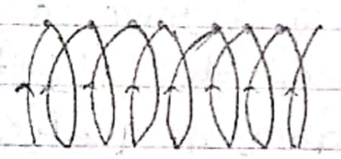
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

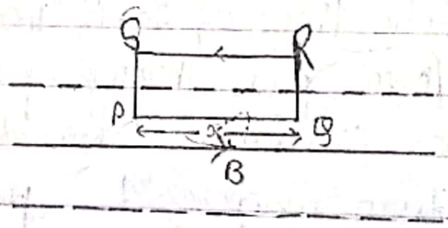
$$\text{Or, } B\, 2\pi a = \mu_0 I$$

$$\text{Or, } \boxed{B = \frac{\mu_0 I}{2\pi a}}$$

which is the required expression of flux density due to infinite long conductor carrying current and is in accordance with Biot and Savart's law. ~~The~~

2) Flux density along the axis of solenoid carrying current.





let us consider a ~~small the~~ uniform solenoid having 'n' number of turns per unit length through which current 'I' is flowing. As the current flows through the solenoid, magnetic field is produced and acts along the axis. ~~let us~~ Let us consider a rectangular magnetic lines of force PQRS due to the small portion of the solenoid and ~~has bee~~ has length 'a' then the line integral of flux density along the closed loop gives the value of flux density due to the solenoid carrying current:

i.e

$$\oint_{PQRSP} \vec{B}\cdot\vec{dl} = \int_P^Q \vec{B}\cdot\vec{dl} + \int_Q^R \vec{B}\cdot\vec{dl} + \int_R^S \vec{B}\cdot\vec{dl} + \int_S^P \vec{B}\cdot\vec{dl} \quad \text{---(1)}$$

Since, the segment QR and SP is $\perp^r$ to the direction of B, $\theta = 90°$

$$\therefore \int_Q^R \vec{B}\cdot\vec{dl} = \int_Q^R B\,dl\cos 90° = 0$$

$$\int_S^P \vec{B}\cdot\vec{dl} = \int_S^P B\,dl\cos 90° = 0$$

As the value of 'B' just outside the solenoid is almost zero.
So,

$$\int_R^S \vec{B}\cdot\vec{dl} = 0$$

And, the segment PQ is parallel to the direction of the field
So, $\theta = 0°$

$$\int_P^Q \vec{B}\cdot\vec{dl} = \int_P^Q B.\,dl\cos 0° = \int_P^Q B\,dl = B\int_P^Q dl$$

$$= Bx$$

$$\therefore \oint_{PQRSP} \vec{B}\cdot\vec{dl} = Bx \quad \text{---(2)}$$

Now,

The total no. of circular coils enclosed by the loop PQRSP,

$(N) = nx$

and, the total current enclosed by the loop PQRSP,

$I' = IN = Inx$

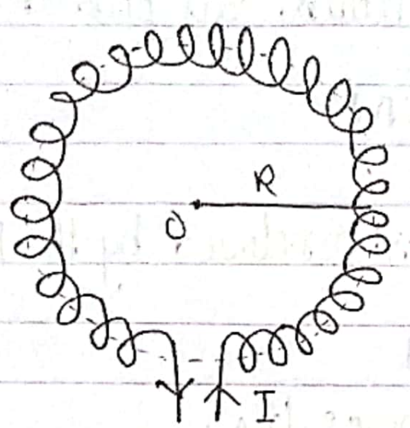
We have from Ampere's Rule,

$$\oint_{PQRSP} \vec{B} \cdot \vec{dl} = \mu_0 I'$$

or, $B\cancel{x} = \mu_0 I n \cancel{x}$

or, $B = \mu_0 n I$

3) Value of flux density due to toroid carrying current.



let us consider a uniform solenoid having 'n' no. of terms ~~through~~ per unit length in which each of the circular coils have the same radii and current I is flowing through them. ~~As the current~~ is given circular shape of radius 'R' to form a toroid. As the current flows through the toroid ~~kinetic~~ magnetic field is produced which is due to magnetic lines of force and is circular. So, the line integral of flux density along the closed loop gives the value of flux density due to the toroid carrying current

ie

$$\oint \vec{B} \cdot \vec{dl} = \oint B\, dl \cos\theta$$

But, $\theta = 0°$ as $\vec{B}$ and $\vec{dl}$ are parallel to each other at that point.

$$\oint \vec{B} \cdot \vec{dl} = \oint B\, dl$$
$$= B \oint dl$$
$$= B \int_0^{2\pi R} dl$$

or, $\oint \vec{B} \cdot \vec{dl} = B 2\pi R$ —— (1)

Now,
the total no. of circular coils enclosed by the loop,

$$(N) = n2\pi R$$

And,
the total current enclosed by the loop.

i.e. $I' = NI$

we have from ampere's law;

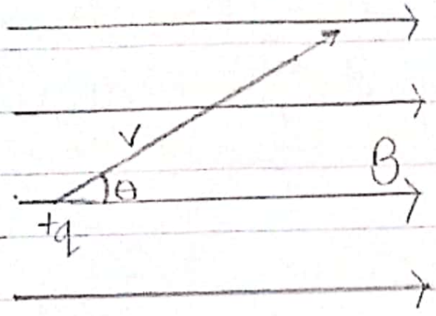
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I'$$

or, $B 2\pi R = \mu_0 NI$

or, $B = \frac{\mu_0 NI}{2\pi R}$

which is the value of flux density due to the toroid carrying current and is in accordance with Biot and Savart's rule.

Force acting on charge moving in presence of magnetic field:



let us consider a charge +q is placed in presence of external magnetic field of strength ~~Q~~ 'B' and is moving with the velocity 'v' making an angle θ with the direction of field. As the charge moves in presence of magnetic field, it experiences force which is known as Lorentz force. The force acting on the charge is—

a) directly proportional to the magnitude of charge

i.e $F \propto q$ — (1)

b) directly proportional to the strength of magnetic field

i.e. $F \propto B$ — (2)

c) directly proportional to velocity with which the charge is moving

i.e. $F \propto V$ — (3)

d) directly proportional to the sine of the angle between the direction of magnetic field and the motion of charge.

i.e $F \propto \sin\theta$ — (4)

Combining all the above relations we get,

$$F \propto qBV \sin\theta$$

$$F = KqBV\sin\theta$$

Where, k is proportionality constant whose value is 1 in S.I. unit

$$\text{or, } F = BqV\sin\theta$$

Since, force is vector quantity, we obtaine the direction of force using flemmings left hand rule. And is expressed in terms of vector notation as

$$\vec{F} = q(\vec{v} \times \vec{B})$$

Special cases:

1) If the charge is moving parallel or anti parallel to the direction of magnetic field

$$\theta = 0^\circ \text{ or } 180^\circ$$

$$\therefore \vec{F} = BqV\sin 0^\circ$$
$$= BqV.0$$
$$= 0$$

2) If the charge is moving perpendicular to the direction of field then,

$$\theta = 90^\circ$$

$$\therefore \vec{F} = BqV\sin 90^\circ$$
$$= BqV. \;//$$

$F = qE$ (Rest)
$\vec{F} = q(\vec{v} \times \vec{B})$
i.e. $\vec{F} \perp^r v$; $\vec{F} \perp^r \vec{B}$
$\vec{v} \propto \vec{d}$

Then,
$\vec{F} \perp^r \vec{d}$
So, $\theta = 90°$
$\therefore W = Fd \cos 90°$
$= 0$

31

When the charge moves being perpendicular to the direction of magnetic field, maximum force acts on it however the work done due to the laurence force is zero. Since ~~at~~ the force acts at 90° work done is zero, the charge begins to move in circular path during which the centripetal force is equal to ~~laurence force~~ lorentz force.

$$\text{or, } \frac{mv^2}{r} = Bqv \sin 90°$$

$$\text{or, } \frac{mv}{r} = Bq$$

$$\text{or, } \frac{m(r\omega)}{r} = Bq \quad [\because v = r\omega]$$

$$\text{or, } m 2\pi f = Bq$$

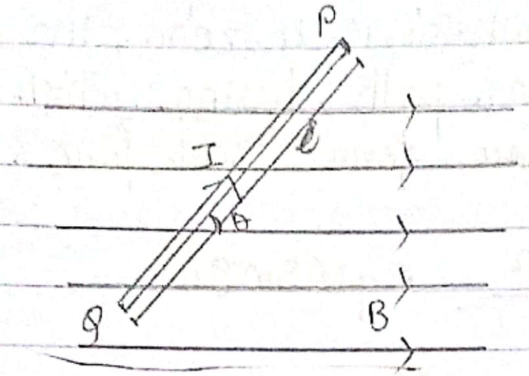
$$\text{or, } f = \frac{Bq}{2\pi m}$$

And,
the time period of revolution

$$T = \frac{1}{f} = \frac{2\pi m}{Bq}$$

Scanned with CamScanner

* Force acting on conductor carrying current placed in presence of magnetic field.



let us consider PQ is a conductor of length 'l' having cross section area 'A' through which current 'I' is flowing and is placed in presence of external magnetic field of strength 'B' making an angle θ with the direction of field. as the current flows through the conductor, the electrons ~~bg~~ begin to move in opposite direction with drift velocity 'v' in opposite direction to that of current flow. Since the electrons move in presence of external magnetic field, they experience force. Hence, the conductor experiences force as well. ~~ther~~
Now, the force acting on electrons as it moves in presence of magnetic field is

$$F = Bqv\sin\theta$$

$$\text{Or, } F = BNev\sin\theta$$

where, N is the no. of free electron ~~is~~
e is the charge of each electron

$$\text{Or, } F = BnVe\,v\sin\theta$$

where, n is the no. of electrons per unit volume

V is the volume of the conductor.

Or, $f = B n A e v_d \sin\theta$ $[\because V = AL]$

Or, $f = BIL \sin\theta$ $[\because I = v_d e n A]$

which is the req. expression of force acting on conductor carrying current placed in presence of external magnetic field. The direction of force acting is obtained using Flemming's left hand rule.

Special cases:

1) If the conductor is placed parallel to the direction of magnetic field, $\theta = 0$

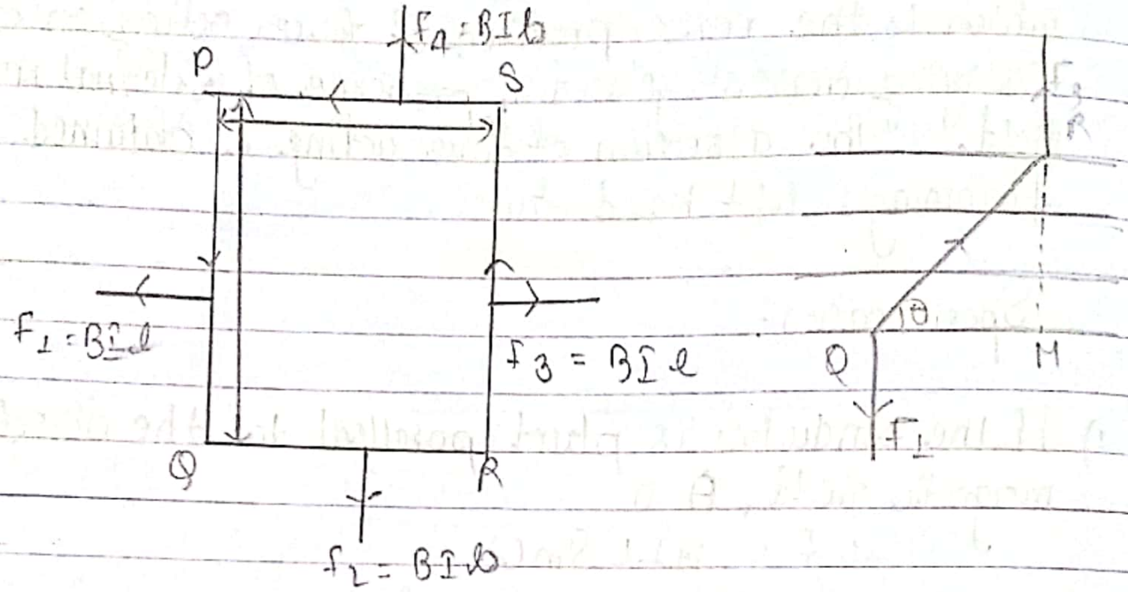
$$\therefore f = BIL \sin 0^\circ$$
$$= 0$$

2) If the conductor is placed perpendicular to the direction of magnetic field, $\theta = 90^\circ$

$$\therefore f = BIL \sin 90^\circ$$
$$= BIL$$

* Torque produced on rectangular coil carrying current placed in presence of magnetic field.

VVI



Let us consider a non-magnetic rectangular frame over which wire is ~~woubd~~ would to form rectangular coil each of area 'A' having length 'l' and breadth 'b' through which current 'I' is flowing. Let B is the strength of the magnetic field in which the plane of the coil is placed.

only for understanding, no need to write in exam

When the plane of the coil is placed perpendicular to the direction of field, the force acting along the segments PQ and RS as well as QR and SP is equal in magnitude opposite in direction and acts along the same line of action. Hence, ~~when~~ torque is not produced.

When the plane of the coil is placed parallel to the direction of field, the force acting along the segment ~~QS or~~ PS and QR is zero. However, the force acting along the segments PQ and RS is equal in magnitude opposite in direction and do not act along the same

line of action. Hence, turning effect of force is observed.
When the plane of the coil makes an angle θ with the direction of field, the force acting along the segments PQ and QR are equal in magnitude opposite in direction and acts along the same line of action which cancels out each other. However, the force acting along the segments PS and RS is equal in magnitude opposite in direction but do not act along the same line ~~and cane~~ cancels out each other producing turning effect of force.

Torque (τ) = Force (F_1) × ⊥ dist. betⁿ the line of action of force

$= F_1 \times QM$

$= BIl \sin 90° \times QR \cos\theta$ [∵ $\sin 90°$ is taken as I ⊥ to B. Ht and $\cos\theta = \frac{QM}{QR}$]

$= BIlb \cos\theta$

$= BIA \cos\theta$

If ~~for~~ there are 'N' no., such rectangular coils then total torque produced

$\tau = BINA \cos\theta$

Special cases:

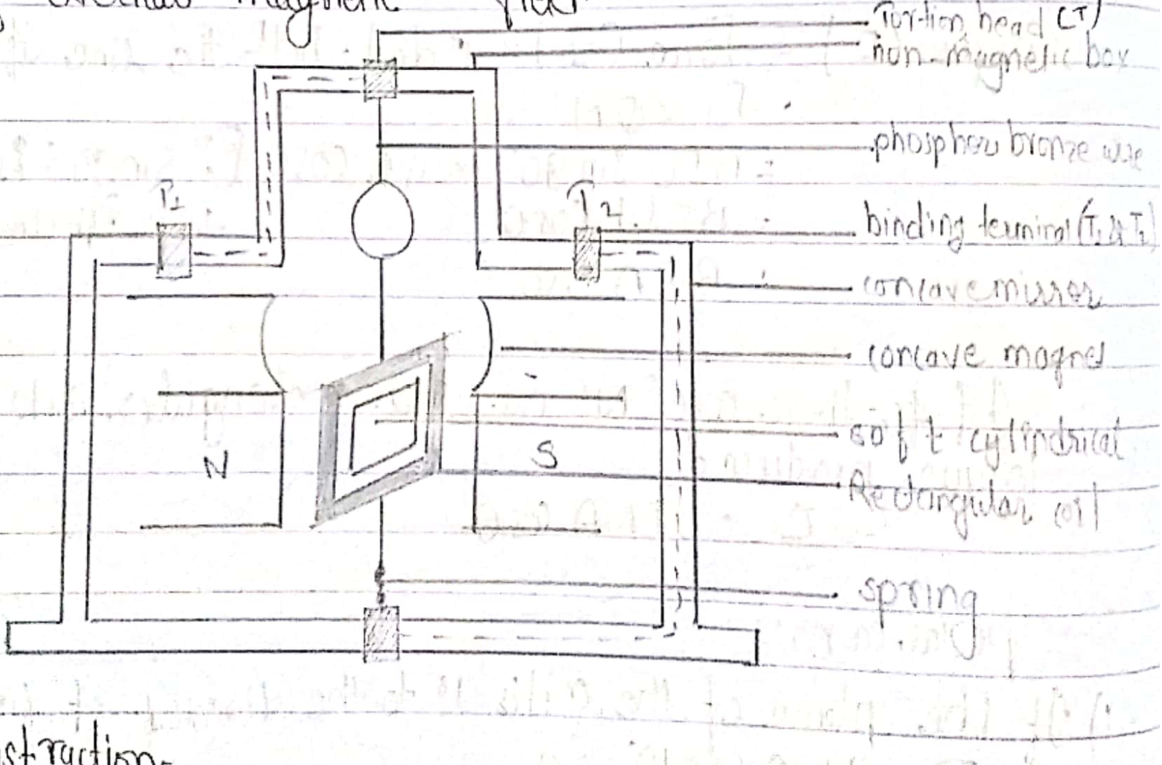
1) If the plane of the coil is ⊥ to the direction of field, θ = 90°.
So, $\tau = BINA \cos 90° = 0$

2) If the plane of the coil is parallel to the direction of field, θ = 0°
So, $\tau = BINA \cos 0° = BINA$

Moving Coil.

Galvanometer is an electrical device which is used to determine the current flowing in the galvanometer and detect the

It works on the principal of torque produce on rectangular coil carrying current placed in presence of external magnetic field.



Constraction-

It consist of a non-magnetic rectangular frame over which wire is wound to form rectangular coil. The plane of coil is placed between the concave magnets. which produces rectangular radial magnetic field such that the plane of the coil is parallel to the direction of magnetic field at all the positions. The upper end of the coil is suspended with the help of phosphor bronze wire from the torsion head (T). A ~~sho~~ soft cylindrical iron core is placed at the middle of rectangular coil which helps to crowd the magnetic lines of force. The lower end of the magnetic coil is attached to the spring made up of the same phospher bronze wire which exerts 'I' restoring torque. A concave mirror is placed which helps to take the reading of needle of galvanometer. The whole set up is placed inside non-magnetic rectangular box as the device is very sensitive. The current is made to flow through the galvanometer by connecting it to the external circuit at binding terminals.

Working-

Let us consider there are 'N' number of rectangular coils in which each of the coils have area 'A' and 'I' is the current flowing through each of the coils in presence of external magnetic field of strength 'B'. As the current flows through the coil, deflection is obtained in the middle of rectangular coil. So, the deflecting torque acting is

$\tau_d = BINA \cos 0°$ $[\theta = 0°]$ as the plane of the coil is parallel to the direction of field at all the positions.

or, $\tau_d = BINA$ ——— (1)

as the plane of the coils get deflected ~~at~~ the needle of galvanometer also gets deflected. So, during this restoring torque acts. ~~The~~ The restoring torque acting is directly proportional to the angle of deflection of needle of galvanometer

i.e. Restoring torque (τ_r) $\propto$ Angle of deflection of needle of galvanomer (α)

or, $\tau_r \propto \alpha$

or, $\tau_r = C\alpha$ — (2) [Where 'C' is the torsional constant of wire] at equilibrium condn.

$$\tau_r = \tau_d$$

Or, $C\alpha = BINA$

Or, $\alpha = \frac{BINA}{C}$

But, B, N, A and C are constant for the given galvanometer

So, $G = \frac{BNA}{C}$

$\therefore \alpha = GI$, where 'G' is the galvanometer constant.

or, $\alpha \propto I$ is the working principle of moving coil galvanometer.

Current Sensitivity (C.S)-

Current sensitivity is defined as the angle of deflection of needle of galvanometer per unit current flowing ~~througcoil~~ through the coil.

i.e. $$C.S = \frac{\alpha}{I} = \frac{\frac{BINA}{c}}{I} = \frac{BNA}{c}$$

Voltage Sensitivity ~~(V)~~ (V.S)-

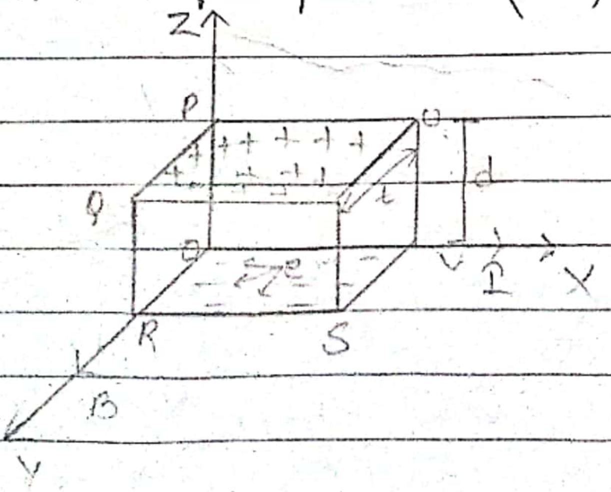
It is defined as angle of deflection of needle of galvanometer per unit potential difference across the coil.

$$V.S = \frac{\alpha}{V} = \frac{\frac{BINA}{c}}{IR} = \frac{BNA}{CR}$$

And the ratio of C.S to V.S gives us the value of resistance offered by the ~~g/c~~ galvanometer.

WI Hall's effect-

The phenomenon in which transverse emf is produced on a conductor carrying current placed in presence of magnetic field acting perpendicular to the current flow is known as Hall's effect. The emf produced during the process is known as Hall's emf or potential (V_H).



let us consider a conductor of cuboid shape having width 'L' and ~~Breadth~~ breadth 'd' is ~~plas~~ placed along positive x-axis which is also the direction of current flow. As the current flows ~~to the~~ c. through the conductor, the electrons begins to move along $-ve$ x-axis with drift velocity (v). The magnetic field of strength 'B' acts along +ve ~~y~~-axis from the back to the front ~~pho~~ face of the conductor. As the electrons begin to move along $-ve$ x-axis ~~it experiences force~~ in presence of magnetic field so it experiences force whose direction is obtained using Flemming's left hand rule. Due to this the electrons get accumulated at the bottom place of the conductor. This induces +ve charge on the upper face of the conductor. Since +ve and $-ve$ charge are apart at a distance 'd', there is a difference in potential which is known as false potential and electric field is also set up along $-ve$ z-axis.

Force acting on electron due to the magnetic field,

$F_m = BeV \sin 90°$ [$\theta = 90°$, & $I \perp^r$ to B]

or, $F_m = BeV$ ——— (1)

Force acting on electron due to Hall's electric field,

$$F_e = eE_H = e\frac{V_H}{d} \text{ ——— (2)}$$

At equilibrium condition

$$F_e = F_m$$

or, $e\dfrac{V_H}{d} = BeV$

or, $\dfrac{V_H}{d} = BV$

or, $\frac{V_H}{d} = \frac{BI}{neA}$ $[\because I = VenA]$

or, $\frac{V_H}{d} = \frac{BI}{netd}$ $[\because A = td]$

or, $V_H = \frac{BI}{net}$ which is req. Hall's emf or potential.

Hall's coefficient (R_H)-

It is defined as the Hall's electric field acting per unit magnetic field strength and per unit current density

i.e $R_H = \frac{E_H}{BJ} = \frac{\frac{V_H}{d}}{\frac{BI}{A}} = \frac{\frac{BI}{net} \times \frac{1}{d}}{\frac{BI}{A}} = \frac{\frac{BI}{neA}}{\frac{BI}{A}} = \frac{1}{ne}$

It's SI unit is m^3/coulomb.

Force acting between two parallel conductors carrying currents.

i) In same direction.

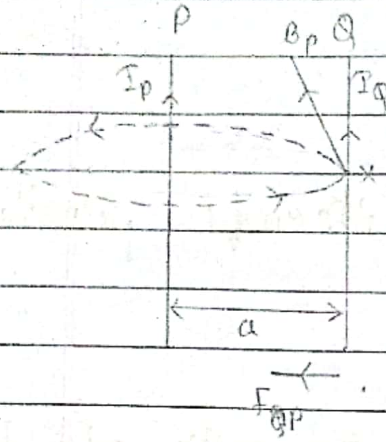


fig-(1)

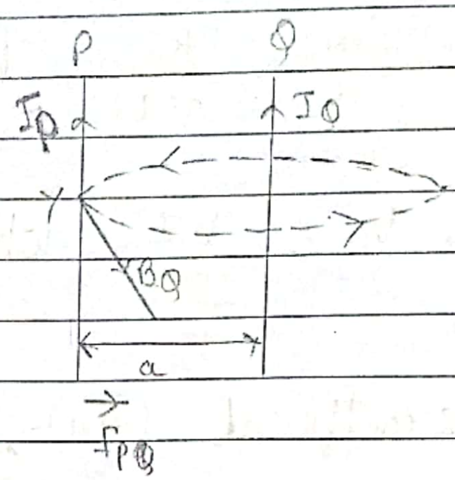


fig-(2)

let us consider P and Q are two ~~types of~~ parallel conductors carrying current I_P and I_Q respectively in same direction. let the conductors are aparted at distance 'a' and l_P and l_Q is the lengths of conductor P and Q respectively.

~~as the~~
As the current flows through conductor 'P', magnetic field is produced. So, the value of magnetic field at point 'R' on the conductor is

$$B_P = \frac{\mu_0 I_P}{2\pi a} \quad \text{——— (1)}$$ (acting $\perp^r$ to I_Q below the place of paper)

Since the current flows through the conductor 'Q' and is present ~~in~~ of magnetic field produced by conductor 'P'. So conductor 'Q' experiences force due to the field of conductor P.

i.e. $F_{QP} = B_P I_Q l_Q \sin 90°$ ($\theta = 90°$ as $B_P \perp^r$ to I_Q)

$$= \frac{\mu_0 I_P I_Q}{2\pi a} l_Q$$ (Acts towards the conductor P)

The direction of force acting on coductor 'Q' ~~due to~~ due to the field of 'P' is obtained using ~~Fe~~ flemming's left hand rule and the force acting per unit length on conductor 'Q' is

$$f_{QP} = \frac{F_{QP}}{l_Q} = \frac{\mu_0 I_P I_Q}{2\pi a} \quad \text{---} (2)$$

Similarly, the force acting per unit length on conductor 'P' due to magnetic field of 'B' is

$$f_{PQ} = \frac{\mu_0 I_P I_Q}{2\pi a} \quad \text{---} (3)$$

and acts towards the conductor 'Q'. Since, this is the simultaneously occuring process the force acting per unit length bet^n the two conductor is attractive in nature.

ii) In opposite direction.

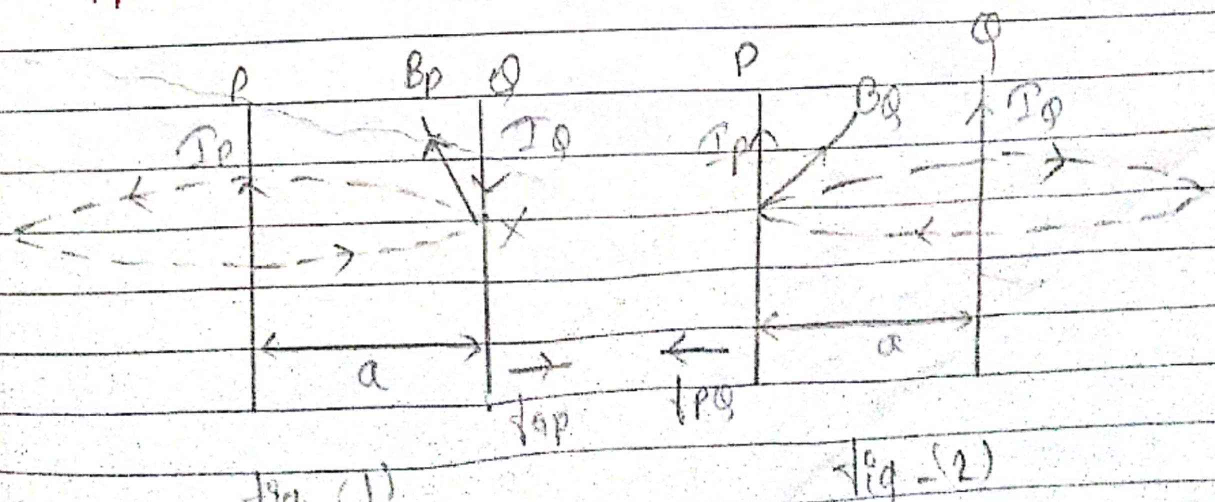


fig-(1) fig-(2)

let us consider P and Q are two parallel conductor carrying current in ~~I~~ I_p and I_Q respectively in opposite direction. let the conductor are aparted at distance 'a' and 'l_p' and 'l_q' be is the length of conductors P and Q respectively.

As the current flows through the conductor 'P'. Magnetic field is produced. so the value of magnetic field at point 'x' on the conductor is

$$B_p = \frac{\mu_0 I_p}{2\pi a}$$

incomplete